International Journal of Mathematics, Statistics and Operations Research
Vol. 1, No. 1, 2021 : pp. 1-9

ARF INDIA
Academic Open Access Publishing
www.arfjournals.com

ITERATIVE THEOREM FOR PRIME DISTRIBUTION

LIAN KE ZOU[1]

Article History

Received : 16 March 2021
Revised : 19 March 2021
Accepted : 7 April 2021
Published : 1 August 2021

ABSTRACT: Based on Mertens theorem, this paper presents and proves the iterative theorem of prime number distribution. A more accurate prime number theorem is obtained by using the iterative theorem of prime number distribution for transformation.

Keywords: prime number; Prime number theorem, prime number distribution iteration theorem, Mertens theorem.

1. Introduction

The prime number theorem is a famous problem in number theory. In 1737, mathematician Leonhard Euler published the Euler product formula at the St. Petersburg Academy of Sciences, which became the theoretical basis for studying the distribution of prime Numbers. In 1798, French mathematician Legendre made a guess based on numerical statistics:

$$\pi(x) \sim \frac{x}{\ln x - 1.08366}, (x \to \infty), \tag{1.1}$$

Here (1.1) is called: prime number theorem. Where $\pi(x)$ is the number of primes not exceeding x.

In 1896, The French mathematician Jacques Hadamard and the Belgian mathematician Charles DE la Valley-Poussin independently proved each other with the abstruse theory of integral functions in the direction of Riemann's complex function theory:[1]

$$\pi(x) \sim \frac{x}{\log x}, (x \to \infty), \tag{1.2}$$

Here (1.2) is the prime number theorem obtained by the joint efforts of many mathematicians.

However, Selberg and Erdös proved the prime number theorem independently by elementary methods.

[1] Chinese Investors Association. 2530104772@qq.com

To cite this article:
Lian Ke Zou, Iterative Theorem for Prime Distribution. *International Journal of Mathematics, Statistics and Operations Research,* 2021. 1(1): 1-9

Mathematicians believe that it is much more difficult to prove the theorem of prime Numbers by elementary methods than by esoteric ones. Elementary proofs of the prime number theorem are particularly difficult to find. Because of this, the work of Selberg and Erdös occupies an important place in number theory.

The elementary proof of prime number theorem is mainly based on Chebyshev inequality, Mertens theorem and Selberg inequality. These are made by Chebyshev function $\Psi(x)$ reasoning get [2]. But the proof is still complicated.

In this paper, proved [3]:

$$\pi(x) \sim \frac{r}{\log x}\frac{1}{1-k}, \quad \frac{\pi(y)}{\pi(x)} = k, \tag{1.3}$$

Then the iteration theorem is obtained:

$$\pi(x) \sim \frac{x}{\log x - \lambda_1}, (x \to \infty), \tag{1.5}$$

Among them

Legendred, a French mathematician, figured out that this is better than (1.2). We also prove that:

Among them $l_1 = 10(1.5)$ Legendred, a French mathematician, figured out that this is better than (1.2). We also prove that:

$$\pi(x) \sim \frac{x}{\log x - \lambda_2}, (x \to \infty), \tag{1.6}$$

Among them:

$$\lambda_2 = 1 + \frac{1}{\log x - 1},$$

(1.6) Better than (1.5). It can also prove that

$$\pi(x) \sim \frac{x}{\log x - \lambda_3}, (x \to \infty), \tag{1.7}$$

and

$$\lambda_3 = 1 + \frac{1}{\log x - 3},$$

(1.7) Better than (1.6). And to prove

$$\pi(x) \sim \frac{x}{\log x - \lambda_4}, (x \to \infty), \tag{1.8}$$

Among them:

$$\lambda_4 = 1 + \frac{1}{\ln x - 3 - 4 / \ln x - 23 / \ln^2 x},$$

(1.8) Better than (1.7).

2. Interval function

Let $x > y$, the prime number of the interval $y \le p \le x$ is p, and the number of $1/p$, that is, the number of the reciprocal of the prime number, is:

$$\pi(x) - \pi(y), \tag{2.1}$$

Due to the $y \le p \le x$, so

$$\frac{1}{x} \le \frac{1}{p} \le \frac{1}{y}, \tag{2.2}$$

by (2.1) and (2.2) It can be confirmed [4] :

$$\frac{1}{x}\left(\pi(x) - \pi(y)\right) \le \sum_{y \le p \le x} \frac{1}{p} \le \frac{1}{y}\left(\pi(x) - \pi(y)\right), \tag{2.3}$$

(2.3) Multiply both sides by x, and you get:

$$\pi(x) - \pi(y) \le x \sum_{y \le p \le x} \frac{1}{p} \le \frac{x}{y}\left(\pi(x) - \pi(y)\right), \tag{2.4}$$

Let's say r is the number of adjacent integers around x. According to (2.4) :

$$\pi(x) - \pi(y) \le x \sum_{y \le p \le x} \frac{1}{p} \le \frac{x}{x - r}\left(\pi(x) - \pi(y)\right), \tag{2.5}$$

By (2.5) You can get

$$\pi(x) - \pi(y) \le x \sum_{y \le p \le x} \frac{1}{p} \le \frac{1}{1 - r / x}\left(\pi(x) - \pi(y)\right), \tag{2.6}$$

For sufficiently large x, let r be small, given by (2.6)

$$\lim_{x \to \infty} \frac{\pi(x) - \pi(y)}{x \sum_{y \le p \le x} \frac{1}{p}} = 1, \tag{2.7}$$

By (2.7) can get:

$$\pi(x) - \pi(y) \sim x \sum_{y \le p \le x} \frac{1}{p}, \tag{2.8}$$

Here (2.8) is called the interval function. Represents the number of primes in the r adjacent integers around a large x.

3. Iteration theorem of prime number distribution

In 1874, mathematician Mertens proved [2] :

$$\lim_{x \to \infty} \sum_{p \le x} \frac{1}{p} - \log\log x = M, \tag{3.1}$$

(2.1) It's called the Mertens theorem. Where the Mertens constant is $M = 0.2614972128476427837554268386086958......$ By (3.1) can get:

$$\lim_{y \to \infty} \sum_{p \le y} \frac{1}{p} - \log\log y = M, \tag{3.1'}$$

From the above two formulas, we can get:

$$x \sum_{y \le p \le x} \frac{1}{p} \sim x(\log\log x - \log\log y) = x \log \frac{\log x}{\log y}, \tag{3.2}$$

Let $y = x - r$, By (3.2) get:

$$x \sum_{y \le p \le x} \frac{1}{p} \sim x \log \frac{\log x}{\log(x-r)} = x \log \frac{\log x}{\log x + \log\left(1 - \frac{r}{x}\right)},$$

Among them

$$\log x + \log\left(1 - \frac{r}{x}\right) = \log x + \log\left(1 - \frac{1}{(x/r)}\right)$$

$$= \log x - \left(\frac{1}{(x/r)} + \frac{1}{2(x/r)^2} + \frac{1}{3(x/r)^3} + ... + \frac{1}{k(x/r)^k}\right),$$

Let r be small, and for sufficiently large x, let the main term $1/(x/r)$, can get

$$\log x - \log(1 - r/x) \sim \log x - r/x,$$

Thus:

$$x\sum_{y\le p\le x}\frac{1}{p}\sim x\log\frac{\log x}{\log x-r/x},$$

That is:

$$x\sum_{y\le p\le x}\frac{1}{p}\sim x\log\frac{1}{1-r/(x\log x)}, \tag{3.3}$$

By (3.4) can get

$$x\log\frac{1}{1-r/(x\log x)}=\frac{r}{\log x}, \tag{3.5}$$

$$\pi(x)-\pi(y)\sim x\sum_{y\le p\le x}\frac{1}{p}\sim\frac{r}{\log x},(x\to\infty), \tag{3.6}$$

For sufficiently large x, there is a system of equations based on (3.6):

$$\pi(x)-\pi(y)\sim\frac{r}{\log x}, \tag{3.7}$$

$$\frac{\pi(y)}{\pi(x)}=k, \tag{3.7'}$$

According to the system of equations (3.7') and (3.8) :

$$\pi(x)\sim\frac{r}{\log x}\frac{1}{1-k}, \tag{3.8}$$

(3.8) Can be called the iterative theorem of prime number distribution.

4. Iterative transformation

For sufficiently large x, it can be obtained from the prime number theorem (1.2) [5] :

$$\pi(x)\sim\frac{x}{\log x},\quad \pi(y)\sim\frac{y}{\log y}, \tag{4.1}$$

By (4.1) get:

$$k=\frac{\pi(y)}{\pi(x)}\sim\frac{y\log x}{x\log y},$$

Substitute into (3.8) and get:

$$\pi(x)\sim\frac{r}{\log x}\frac{1}{1-\dfrac{y\log x}{x\log y}}=\frac{r}{\log x}\frac{x\ln y}{x\log y-y\log x}, \tag{4.2}$$

Let $y = x - r$, By (4.2) can get:

$$\pi(x) \sim \frac{r}{\log x} \frac{x \log(x-r)}{x \log(x-r) - (x-r)\log x}$$

$$= \frac{r}{\log x} \frac{x \log\left(x(1-r/x)\right)}{x \log\left(x(1-r/x)\right) - x\log x + r \log x}$$

$$= \frac{r}{\log x} \frac{x \log x + x\log(1-r/x)}{x \log x + x\log(1-r/x) - x\log x + r \log x}$$

$$= \frac{r}{\log x} \frac{x \log x - r}{x \log x - r - x\log x + r \log x},$$

The resulting

$$\pi(x) \sim \frac{1}{\log x} \frac{x \log x - r}{\log x - 1} = \frac{x - r/\log x}{\log x - 1},$$

That is:

$$\pi(x) \sim \frac{x}{\log x - 1} - \frac{r}{\log x(\log x - 1)},$$

Let r be very small. For sufficiently large x, the main term $x/\log x - 1)$, is taken to obtain [7] :

$$\pi(x) \sim \frac{x}{\log x - 1},$$

That is:

$$\pi(x) \sim \frac{x}{\log x - \lambda_1}, \tag{4.3}$$

Among them $\lambda_1 = 1$, formula (1.5) of the iterative theorem for prime Numbers is thus proved.

And we can continue with the transformation, From (4.3) gr:

$$\pi(x) \sim \frac{x}{\log x - \lambda_1}, \quad \pi(y) \sim \frac{y}{\log y - 1},$$

There are:

$$k = \frac{\pi(y)}{\pi(x)} \sim \frac{y(\log x - 1)}{x(\log y - 1)}, \tag{4.4}$$

Substitute (4.4) into (3.8) to get:

$$\pi(x) \sim \frac{r}{\log x} \frac{1}{1 - \dfrac{y(\log x - 1)}{x(\log y - 1)}} = \frac{r}{\log x} \frac{x \log y - x}{x \log y - x - y \log x + y},$$

Let $y = x - r$ get :

$$\pi(x) \sim \frac{r}{\log x} \frac{x \log(x - r) - x}{x \log(x - r) - x - (x - r) \log x + x - r}$$

$$= \frac{r}{\log x} \frac{x \log x - r - x}{x \log x - r - x - x \log x + r \log x + x - r}$$

$$= \frac{1}{\log x} \frac{x(\log x - 1 - r / x)}{\log x - 2}$$

$$= \frac{x(\log x - 1)}{\log x(\log x - 2)} - \frac{r}{\log x(\log x - 2)},$$

Let r be small, and for sufficiently large x, the main term is taken to obtain:

$$\pi(x) \sim \frac{x(\log x - 1)}{\log x(\log x - 2)}, \tag{4.5}$$

Let

$$a(x) = \frac{\log x - 2}{\log x - 1},$$

From (4.5) can get

$$\frac{x(\log x - 1)}{\log x(\log x - 2)} = \frac{x}{a(x) \log x}$$

$$= \frac{x}{\log x - \log x + a(x) \log x}$$

$$= \frac{x}{\log x - \log x + a(x) \log x}$$

$$= \frac{x}{\log x - \dfrac{\log x}{\log x - 1}},$$

That is:

$$\pi(x) \sim \frac{x}{\log x - \lambda_2}, \tag{4.6}$$

Among them

$$\lambda_2 = 1 + \frac{1}{\log x - 1},$$

Here the intensity of (4.6) exceeds the (1.1) Legendred conjecture.
You can keep going, and get a more exact prime theorem.
from (4.6) The transformation will give you [6].

$$\pi(x) \sim \frac{x}{\log x - \lambda_3}, \tag{4.7}$$

Among them

$$\lambda_3 = 1 + \frac{1}{\log x - 3},$$

The transformation of (4.7) can be obtained [8]

$$\pi(x) \sim \frac{x}{\log x - \lambda_4}, \tag{4.8}$$

Among them

$$\lambda_4 = 1 + \frac{1}{\ln x - 3 - 4/\ln x - 23/\ln^2 x},$$

For the value 1, the calculation is as follows:

x,	$\pi(x)$,	λ_1,	λ_2,
10^{12} ,	37607912018,	37550193650,	37603214823ÿ
10^{18},	24739954287740860,	24723998785919975,	24739121219851663,
x,	$\pi(x)$,	λ_3,	λ_4,
10^{12},	37607912018,	37607526632,	37607937261ÿ
10^{18},	24739954287740860,	24739908399573115,	24739954036830066,

again transformation will lead to a more accurate prime number theorem.

Acknowledgement

Thanks for Department of Theoretical Physics and Pure Mathematics Fuzhou Institute of Original Physics Xiao Chuen Mei's examination and adaptation.

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